VMC Vidyamandir Classes SINCE 1986
IIT JEE | MEDICAL | FOUNDATION

CODE A

JEE Main Home Practice Test - 4 | JEE - 2024

Date: 6/01/2024 **Maximum Marks: 300**

Timing: 10:00 AM to 1:00 PM **Duration : 3.0 Hours**

General Instructions

1. The test is of **3 hours** duration and the maximum marks is **300.**
2. The question paper consists of **3 Parts** (Part I: **Physics**, Part II: **Chemistry**, Part III: **Mathematics**). Each Part has **two** sections (Section 1 & Section 2).
3. **Section 1** contains **20 Multiple Choice Questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.
4. **Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. You will NOT be allowed to attempt the sixth question. If you wish to attempt any other question apart from the five already attempted, then you will have to delete any one response from the five previously answered and then proceed to answer the new one.**
   The answer to each question should be **rounded off to the nearest integer**.
5. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.
6. On completion of the test, the candidate must hand over the Answer Sheet to the **Invigilator** on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them**.

Marking Scheme

1. **Section – 1:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.
2. **Section – 2:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.

Syllabus

Physics: Full Syllabus
Chemistry: Full Syllabus
Mathematics: Full Syllabus

Name of the Candidate (In CAPITALS) : _____

Roll Number : _____

OMR Bar Code Number : _____

Candidate's Signature : _____ **Invigilator's Signature** _____

PART - I : PHYSICS 100 MARKS

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE CHOICE is correct.

1. A ray beam of width 1 cm is incident from air to air-water boundary at an angle of 45°. What is the width of the refracted beam in water? (Refractive index of water $=\sqrt{2}$)

(A) $\frac{1}{2}$cm **(B)** $\frac{1}{\sqrt{2}}$ cm **(C)** 1 cm **(D)** $\frac{\sqrt{3}}{\sqrt{2}}$ cm

2. Find the energy released in the reaction ${}_1H^1 + {}_3Li^7 \rightarrow 2\,{}_2He^4$

(Given: Binding energy per nucleon of ${}_3Li^7 = 5.6$ MeV, Binding energy per nucleon of ${}_2He^4 = 7.06$ MeV)

(A) 30.5 MeV **(B)** 17.3 MeV **(C)** 22.4 MeV **(D)** 12.8 MeV

3. A neutral atom which is stationary at the origin in a gravity free space emits an α-particle *A* in the *z*-direction. The product atom is *P*. A uniform magnetic field exists in the *z*-direction. Disregarding the electromagnetic forces between *A* and *P*, which of the following is correct?

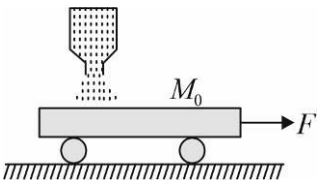
(A) *A* and *P* will move along circular paths of equal radii

(B) *A* has greater time period of rotation than *P*

(C) *A* has greater kinetic energy than *P*

(D) *A* and *P* will meet again somewhere in the *yz*-plane

4. In the figure, a cart moves on a smooth horizontal surface due to an external constant force of magnitude *F*. The initial mass of the cart is M_0 and velocity is zero. Sand falls on to the cart with negligible velocity at constant rate μ kg/s and sticks to the cart. The velocity of the cart at time *t* is:



(A) $\frac{Ft}{M_0 + \mu t}$ **(B)** $\frac{Ft}{M_0}e^{\mu t}$ **(C)** $\frac{Ft}{M_0}$ **(D)** $\frac{Ft}{M_0 + \mu t}e^{\mu t}$

Space for Rought Work

5. A particle of mass 1kg is performing SHM with maximum kinetic energy 2J. The average speed of particle during the interval of time in which it moves from one extreme position to the other extreme position is:

(A) $\frac{2}{\pi}$ m/s **(B)** $\frac{4}{\pi}$ m/s **(C)** 1 m/s **(D)** 0.5 m/s

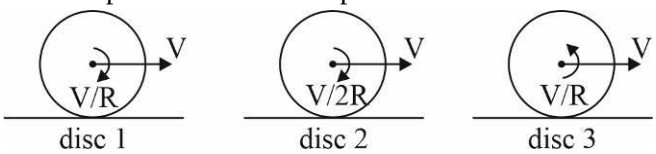
6. In a hydrogen like atom the energy required to excite the electron from 2^{nd} to 3^{rd} orbit is 47.2 eV. What is the atomic number of the atom?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5

7. Two hollow spheres of same thickness are filled with ice. The ratio of their diameter is 1 : 2 and the ratio of thermal conductivities of the materials is 2 : 3. The ratio of times in which the ice gets melted in the two spheres is :

(A) 3 : 4 **(B)** 4 : 3 **(C)** 3 : 8 **(D)** 8 : 3

8. Three discs are performing rolling motion on frictionless ground as shown in diagram. Find out in which disc, more number of particles will have speed less than *V*.



(A) disc 1 = disc 3 > disc 2 **(B)** disc 1 = disc 3 < disc 2

(C) disc 1 = disc 3 = disc 2 **(D)** disc 1 > disc 3 > disc 2

9. The equation E = *pc* is valid (here E is energy, P is momentum and c is speed of light in vacuum) :

(A) For an electron as well as for a photon **(B)** For an electron but not for a photon

(C) For a photon but not for an electron **(D)** Neither for an electron nor for a photon

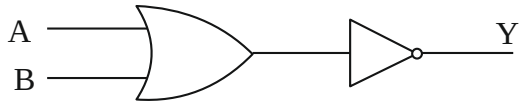
10. A car is moving with acceleration a starting from rest along a circular track of radius *R*, if friction coefficient is μ, then at any time *t* car will be moving without skidding if :

(A) $\mu mg \geq \frac{ma^2t^2}{R}$ **(B)** $\mu g \geq a$

(C) $\mu g \geq \sqrt{a^2 + \left(\frac{a^2t^2}{R}\right)^2}$ **(D)** $\mu g \geq \sqrt{a^2 - \left(\frac{a^2t^2}{R}\right)^2}$

Space for Rought Work

11. The given circuit is for the gate :



(A) NOR **(B)** NAND **(C)** NOT **(D)** XOR

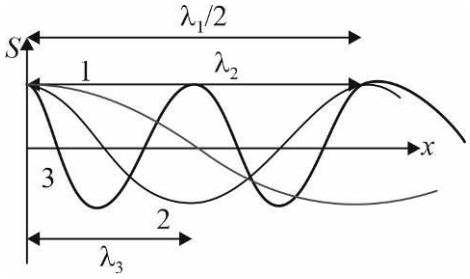
12. A particle starts moving along horizontal line with constant acceleration. After t_1 second the acceleration reverse its direction without changing the magnitude. Find the total time, when the particle returns to its starting point:

(A) $2t_1$ **(B)** $4t_1$ **(C)** $(\sqrt{2}+1)t_1$ **(D)** $(\sqrt{2}+2)t_1$

13. Assuming the xylem tissues through which water rises from root to the branches in a tree to be of uniform cross-section, find then maximum radius of xylem tube in a 10 m high coconut tree so that water can rise to the top. (Surface tension of water = 0.1 N/m, Angle of contact of water with xylem tube = 60°) :

(A) 1 cm **(B)** 1 mm **(C)** 10 μm **(D)** 1 μm

14. Figure shown is a graph, at a certain time *t*, of the displacement function $S(x, t)$ of three sound waves 1, 2 and 3 as marked on the curves that travel along *x*-axis through air. If P_1, P_2 and P_3 represent their pressure amplitudes respectively, then correct relation between them is:



(A) $P_1 > P_2 > P_3$ **(B)** $P_3 > P_2 > P_1$ **(C)** $P_1 = P_2 = P_3$ **(D)** $P_2 > P_3 > P_1$

15. The electromagnetic wave travels in a medium at a speed of 2.0×10^8 *m/s*. The relative permeability of the medium is 1.0. The relative permittivity of the medium will be:

(A) 2.25 **(B)** 4.25 **(C)** 6.25 **(D)** 8.25

Space for Rought Work

16. An alternating current is given by the equation $i = i_1 \cos \omega t + i_2 \sin \omega t$. The r.m.s. current is given by:

(A) $\frac{1}{\sqrt{2}}(i_1 + i_2)$ **(B)** $\frac{1}{\sqrt{2}}(i_1 + i_2)^2$ **(C)** $\frac{1}{\sqrt{2}}(i_1^2 + i_2^2)^{1/2}$ **(D)** $\frac{1}{2}(i_1^2 + i_2^2)^{1/2}$

17. There is a current of 20 amperes in a copper wire of 10^{-6} square metre area of cross-section. If the number of free electrons per cubic metre is 10^{29}, then the drift velocity is:

(A) $125 \times 10^{-3}\ m/\sec$ **(B)** $12.5 \times 10^{-3}\ m/\sec$

(C) $1.25 \times 10^{-3}\ m/\sec$ **(D)** $1.25 \times 10^{-4}\ m/\sec$

18. An inductor *L* and a resistor *R* are connected in series with a direct current source of emf *E*. The maximum rate at which energy is stored in the magnetic field is:

(A) $\frac{E^2}{4R}$ **(B)** $\frac{E^2}{R}$ **(C)** $\frac{4E^2}{R}$ **(D)** $\frac{2E^2}{R}$

19. Least count of vernier callipers is 0.01 cm. When the two jaws of the instrument touch each other the 5th division of the vernier scale coincide with a main scale division and the zero of the vernier scale lies to the left of the zero of the main scale. Furthermore, while measuring the diameter of a sphere, the zero mark of the vernier scale lies between 2.4 cm and 2.5 cm and the 6th vernier division coincides with a main scale division. Calculate the diameter of the sphere.

(A) 2.51 cm **(B)** 2.46 cm **(C)** 2.40 cm **(D)** 2.41 cm

20. An ideal gas is taken through a cyclic thermo dynamical process through four steps. The amounts of heat involved in these steps are:

Q_1 = 5960 Joules; Q_2 = – 5585 joules: Q_3 = – 2980 joules; Q_4 = 3645 joules

Respectively and works involved are W_1 = 2200 Joules; W_2 = – 825 Joules; W_3 = – 1100 Joules and W_4 respectively. The value of W_4 is:

(A) 1315 Joules **(B)** 275 Joules **(C)** 765 Joules **(D)** 675 Joules

Space for Rought Work

SECTION-2

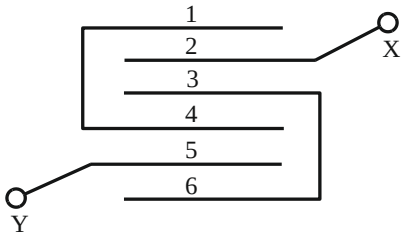
Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

1. A small potassium foil is placed (perpendicular to the direction of incidence of light) a distance r = 0.5 m from a point light source whose output power P_0 is 1.0W. Assuming wave nature of light, how long (in seconds) would it take for the foil to soak up enough energy (= 1.8 eV) from the beam to eject an electron? Assume that the ejected photoelectron collected its energy from a circular area of the foil whose radius equals the radius of a potassium atom $(1.3\times10^{-10}\,\text{m})$.

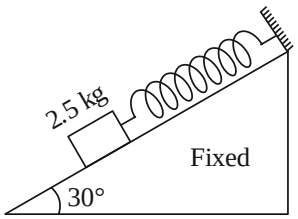
2. A transmitting antenna at the top of a tower has a height of 36 m and the height of the receiving antenna is 49 m. The maximum distance between them, for satisfactory communication in the LOS mode is d. (Radius of earth = 6400 km) find x if $x=\frac{d}{15.5}km.$

3. Two planets A and B have the same material density. If the radius of A is twice that of B, then the ratio of the escape velocity from the surface $\frac{v_A}{v_B}$ is _____ .

4. 6 identical plates are arranged as shown in figure The equivalent capacitance between Y and X is $X\frac{\varepsilon_0 A}{d}$ the value of X is _____ .

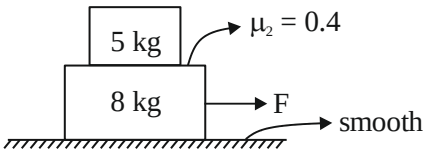


5. A smooth inclined plane having angle of inclination 30° with horizontal has a mass 2.5 kg held by a spring which is fixed at the upper end. If the mass is taken 1.25 cm up along the surface of the inclined plane, the tension in the spring reduces to zero. If the mass is then released, the angular frequency of oscillation in radian per second is _____ . $\left[\text{Take } g=10\,m/s^2\right]$

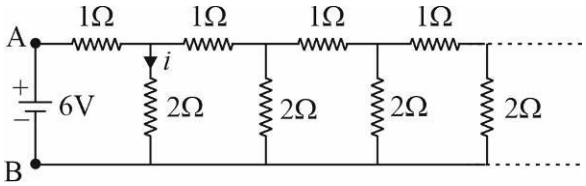


Space for Rought Work

6. Find the maximum possible force (in Newton) which can be applied to the 8 kg block shown in figure to move both the blocks together if bottom surface is frictionless. [Take $g = 10\ \text{m/s}^2$]



7. An infinite ladder network of resistances is constructed with 1 ohm and 2 ohm resistances. The 6 V battery between A and B has negligible internal resistance. Current through the marked 2Ω resistance is *i* Amperes. Value of $10i$ is _____.



8. A battery of 100 V is connected to a series combination of two similar parallel plate capacitors. If dielectric of dielectric constant 4 is inserted between the plates of second capacitor, then the potential difference across the first capacitor will become _____ (in volts).

9. A solid sphere and a ring both having same mass and radius are placed at the top of an rough inclined plane and released. The friction coefficient between the objects and the incline are same but not sufficient to allow rolling. the ratio of time to reach the bottom $\left(\frac{t_{\text{sphere}}}{t_{\text{ring}}}\right)$ is _____ .

10. A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first 2 sec, it rotates through an angle θ_1. In the next 2 sec, it rotates through an additional angle θ_2. The ratio of θ_2 / θ_1 is _____ .

Space for Rought Work

PART - II : CHEMISTRY 100 MARKS

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE CHOICE is correct.

1. Acetamide is treated separately with the following reagents. Which one of these give methylamine?

(A) PCl_5 **(B)** Sodalime

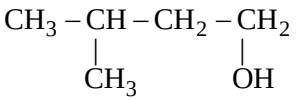
(C) $NaOH + Br_2$ **(D)** Hot concentrated H_2SO_4

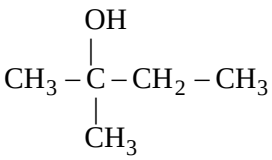
2. $KMnO_4$ is a strong oxidizing agent in acid medium. To provide acid medium, H_2SO_4 is used instead of HCl. This is because :

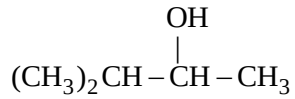
(A) H_2SO_4 is a stronger acid than HCl **(B)** HCl is oxidized by $KMnO_4$ to Cl_2

(C) H_2SO_4 is a dibasic acid **(D)** Rate is faster in the presence of H_2SO_4

3. Arrange the following alkanols, 1, 2 and 3 in order of their reactivity towards acid catalysed dehydration.

(1) 

(2) 

(3) 

(A) $1 > 2 > 3$ **(B)** $2 > 1 > 3$ **(C)** $2 > 3 > 1$ **(D)** $3 > 2 > 1$

4. Given below are two statements

Statement-I: the pentavalent oxide of group –15 element E_2O_5, is less acidic than trivalent oxide E_2O_3 of the same element

Statement-II: The acidic character of trivalent oxide of group 15 element E_2O_3, decreases down the group.

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false, but Statement II is true

Space for Rought Work

5. For the electrochemical cell, $M|M^+||X^-|X$, $E°(M^+/M) = 0.44V$ and $E°(X/X^-) = 0.33V$. From this data, it can be deduce that:

(A) $M + X \longrightarrow M^+ + X^-$ is the spontaneous reaction

(B) $M^+ + X^- \longrightarrow M + X$ is the spontaneous reaction

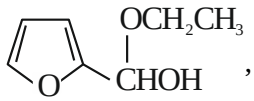
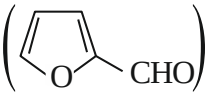
(C) $E_{cell} = 0.77V$

(D) $E_{cell} = -0.77V$

6. The electronegativity order of O, F, Cl and Br is:

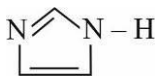
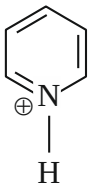
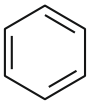
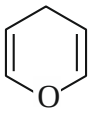
(A) F > O > Cl > Br **(B)** F > Cl < Br > O

(C) Br > Cl > F > O **(D)** F < Cl < Br < O

7. Compound  , formed by the reaction of furfural  with ethanol is:

(A) an aldol **(B)** an acetal **(C)** a ketal **(D)** a hemiacetal

8. Identify the compound which is not aromatic:

(A)  **(B)**  **(C)**  **(D)** 

9. In Lassaigne's test, a blood red colouration indicates the presence of:

(A) Nitrogen **(B)** Sulphur

(C) Both nitrogen and sulphur **(D)** Both nitrogen and halogens

10. The correct order of boiling point of hydrides of group 16 elements is

(A) $H_2S < H_2Se < H_2Te < H_2O$ **(B)** $H_2O < H_2S < H_2Se < H_2Te$

(C) $H_2S < H_2Te < H_2Se < H_2O$ **(D)** $H_2Se < H_2S < H_2Te < H_2O$

Space for Rought Work

11. The correct increasing order of bond angle is:

(A) $BF_3 < NF_3 < PF_3 < ClF_3$ **(B)** $ClF_3 < PF_3 < NF_3 < BF_3$

(C) $BF_3 = NH_3 < PF_3 < ClF_3$ **(D)** $BF_3 < NH_3 < PF_3 > ClF_3$

12. Which of the following has more number of unpaired electron?

(A) Zn^+ **(B)** Fe^{2+} **(C)** Ni^{2+} **(D)** Cu^+

13. Which of the following is/are correct

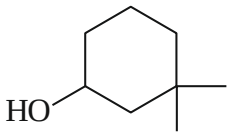
I. Lanthanides forms type M_2O_3 oxides and $M(OH)_3$ hydroxides

II. lanthanides forms LnC_3, LnC_2, LnC type carbides

III. All lanthanide ions are paramagnetic

IV. Misch metal is used in making of bullets, shells

(A) I and II **(B)** II and III **(C)** I and IV **(D)** All

14. The IUPAC name of the compound  is:

(A) 1, 1-Dimethyl-3-cyclohexanol **(B)** 1, 1-Dimethyl-3-hydroxycylohexane

(C) 3, 3-Dimethyl-1-cyclohexanol **(D)** 3, 3-Dimethyl-1-hydroxycyclohexane

15. Which one of the following is most reactive towards nucleophilic substitution reaction?

(A) $CH_2 = CH - Cl$ **(B)** C_6H_5Cl

(C) $CH_3CH = CH - Cl$ **(D)** $ClCH_2 - CH = CH_2$

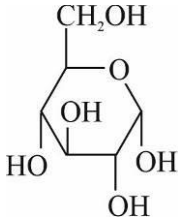
16. Because of lanthanoid contraction, which of the following pairs of elements have nearly same atomic radii? (Number in the brackets are atomic numbers).

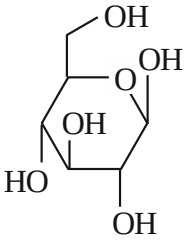
(A) Zr(40) and Nb(41) **(B)** Zr(40) and Hf(72)

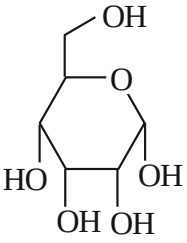
(C) Zr(40) and Ta(73) **(D)** Ti(22) and Zr(40)

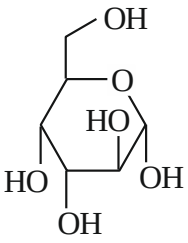
Space for Rought Work

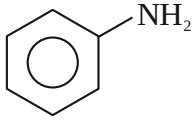
17. Which of the following structures represents α-D- glucopyranose?

(A) 

(B) 

(C) 

(D) 

18.  $\xrightarrow[\Delta]{CHCl_3,\ KOH} X \xrightarrow[H_2O]{LiAlH_4} Y$; (Y) is:

(A) $Ph-\underset{\displaystyle H}{\underset{|}{N}}-CH_3$

(B) $Ph-CH_2-NH_2$

(C) $Ph-\underset{\displaystyle H}{\underset{|}{N}}-\overset{\displaystyle O}{\overset{\|}{C}}-H$

(D) $Ph-\overset{\oplus}{N}\equiv\overset{\ominus}{C}$

19. The ratio of dissociation constants of two weak acids HA and HB is 4. At what molar concentration ratio, the two acids will have same pH?

(A) 2 **(B)** 0.5 **(C)** 4 **(D)** 0.25

20. Which molecule is T-shaped?

(A) BeF_2 **(B)** BCl_3 **(C)** NH_3 **(D)** ClF_3

Space for Rought Work

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

1. Calculate the value of A. $A = \frac{E_{1,2}}{2E_{2,1}}$ where $E_{n,z}$ = Energy of electron in n^{th} orbit, Z = atomic number of hydrogen like species.

2. HCl gas is passed into water, yielding a solution of, density 1.095 g mL^{-1} and containing 30% HCl by weight. Calculate the molarity of the solution.

3. The polymerisation of propene to linear polypropene is represented by the reaction

$$n\left(\begin{array}{l} CH_3 \\ | \\ CH=CH_2 \end{array}\right) \longrightarrow \left(\begin{array}{l} CH_3 \\ | \\ -CH-CH_2- \end{array}\right)_n$$

where *n* has large integral value, the average enthalpies of bond dissociation for $(C=C)$ and $(C-C)$ at 298 K are + 590 and + 331 kJ mol^{-1} respectively. The enthalpy of polymerisation per mole of propylene is –x kJ/mole. Then value of $\frac{x}{36}$ is_____.

4. The rate of decomposition of $NH_3(g)$ at 10 atm on platinum surface is zero order. What is rate of formation (in M min^{-1}) of $H_2(g)$, if rate constant of reaction $2NH_{3(g)} \longrightarrow N_2(g) + 3H_2(g)$ is 2.0 M min^{-1}?

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5. The vapour pressure of a liquid solution containing A and B is 99 torr. Calculate mole % of B in vapour phase. (Given: $P°_A = 100$ torr; $P°_B = 80$ torr)

6. Balance the following reaction and determine the value of $\frac{a}{e}$

$$aCl_2 + bOH^- \longrightarrow cClO_3^- + dCl^- + eH_2O$$

7. How much amount of white precipitate formed by treating 0.1 moles of $PtCl_2.2NH_3$ with excess of $AgNO_3$?

8. A sample of 28 mL of H_2O_2 (aq) solution required 10 mL of 0.1 M $KMnO_4$ (aq) solution for complete reaction in acidic medium. What is the volume strength of H_2O_2?

9. If solubility of AgCl in 0.2 M solution of $AgNO_3$ is represented as $y \times 10^{-10}$ then find the value of *y*. (Given: $K_{sp(AgCl)} = 10^{-10}$)

10. How many faradays are required for reduction of 1 mole $C_6H_5NO_2$ into $C_6H_5NH_2$?

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PART - III : MATHEMATICS 100 MARKS

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE CHOICE is correct.

1. Let $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b}\times\vec{c}=\vec{b}\times\vec{a}$ and $\vec{c}.\vec{a}=0$, the $\vec{c}.\vec{b}$ is equal to:

(A) $\frac{1}{2}$ **(B)** $-\frac{3}{2}$ **(C)** $-\frac{1}{2}$ **(D)** -1

2. Two dices are rolled. If both dices have six faces numbered 1, 2, 3, 5, 7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 7 is:

(A) $\frac{5}{12}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{13}{36}$

3. The circles $x^2+y^2+2ax+c^2=0$ and $x^2+y^2+2by+c^2=0$ touch each other if?

(A) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$ **(B)** $\frac{1}{a^2}-\frac{1}{b^2}=\frac{1}{c^2}$ **(C)** $b=\frac{2ac}{a+c}$ **(D)** None of these

4. If $A=\begin{pmatrix}0 & \sin\alpha\\ \sin\alpha & 0\end{pmatrix}$ and $\det\left(A^2-I\right)=0$, then a possible value of α is:

(A) $\frac{\pi}{2}$ **(B)** $\frac{\pi}{6}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{\pi}{4}$

5. The equation $x^2-4x+[x]+3=x[x]$, where $[x]$ denotes the greatest integer function, has:

(A) A unique solution in $(-\infty, 1)$ **(B)** Exactly two solution in $(-\infty, \infty)$

(C) No solution **(D)** A unique solution in $(-\infty, \infty)$

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6 A group of students comprises of 5 boys and *n* girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then *n* is equal to:

(A) 28 **(B)** 27 **(C)** 25 **(D)** 24

7. The 4th term of GP is 500 and its common ratio is $\frac{1}{m}$, $m \in N$. Let S_n denote the sum of the first *n* terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of *m* is _____.

(A) 10 **(B)** 12 **(C)** 14 **(D)** None of these

8. The area of the triangle with vertices $A(z), B(wz)$ & $C(z+wz)$ is:

where *w* is imaginary cube root of unity.

(A) $\frac{3\sqrt{3}}{4}|z+wz|^2$ **(B)** 1 **(C)** $\frac{\sqrt{3}}{4}|z|^2$ **(D)** $\frac{1}{2}|z|^2$

9. Let $z_1 = 2+3i$ and $z_2 = 3+4i$. The set $S = \left\{ z \in C : |z-z_1|^2 - |z-z_2|^2 = |z_1 - z_2|^2 \right\}$ represents a:

(A) Straight line with the sum of its intercepts on the coordinate axes equals –18

(B) Hyperbola with eccentricity 2

(C) Hyperbola with the length of the transverse axis 7

(D) Straight line with the sum of its intercepts on the coordinates axes equals 14

10. Let $f(x) = \frac{\sin^{-1}(1-\{x\}) \times \cos^{-1}(1-\{x\})}{\sqrt{2\{x\}} \times (1-\{x\})}$, where $\{x\}$ denotes the fractional part of *x*:

$R = \lim\limits_{x \to 0^+} f(x)$ is equal to:

(A) $\frac{\pi}{2}$ **(B)** $\frac{\pi}{2\sqrt{2}}$ **(C)** $\frac{\pi}{\sqrt{2}}$ **(D)** $\sqrt{2}\pi$

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11. In triangle ABC, equation of side BC is $x-y=0$. Circumcentre and orthocentre of the triangle are (2, 3) and (5, 8), respectively. Equation of circumcircle of the triangle is:

(A) $x^2+y^2-4x-6y-21=0$ **(B)** $x^2+y^2-4x-6y-27=0$

(C) $x^2+y^2+4x+6y-27=0$ **(D)** $x^2+y^2-4x-6y+27=0$

12. Evaluate $I=\int_{-\pi}^{\pi}\frac{x\sin x}{e^x+1}dx$

(A) 2π **(B)** π **(C)** 0 **(D)** $\frac{\pi}{2}$

13. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set A × B, each having at least three elements is :

(A) 510 **(B)** 219 **(C)** 256 **(D)** 275

14. Let *a*, *b* be two real numbers such that *ab* < 0. If the complex number $\frac{1+ai}{b+i}$ is of unit modulus and *a* + *ib* lies on the circle |*z* – 1| = |2*z*|, then a possible value of $\frac{1+[a]}{4b}$, where [*t*] is greatest integer function, is:

(A) 1 **(B)** $-\frac{1}{2}$ **(C)** $\frac{1}{2}$ **(D)** None of these

15. If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$ and $y(0)=1$, then $y\left(\frac{\pi}{2}\right)$ is equal to:

(A) $\frac{4}{3}$ **(B)** $\frac{1}{3}$ **(C)** $-\frac{2}{3}$ **(D)** $-\frac{1}{3}$

16. The value of *x* in the expression of $(x+x^{\log_{10}x})^5$, if the third term in the expansion is 10^6. (where *x* > 0)

(A) $5, 10^{-2}$ **(B)** $8, 10^{-5}$ **(C)** $10, 10^{-5/2}$ **(D)** $15, 10^{-2/5}$

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17. If $\tan^{-1}\left(x+\frac{3}{x}\right)-\tan^{-1}\left(x-\frac{3}{x}\right)=\tan^{-1}\frac{6}{x}$, then the value of $5x^8-4x^4+7$ equals:

(A) 397 **(B)** 393 **(C)** 376 **(D)** None of these

18. Let $f(x)=x+\cfrac{1}{2x+\cfrac{1}{2x+\cfrac{1}{2x+....\infty}}}$

the value of $f(50)\cdot f'(50)$, is:

(A) 25 **(B)** 0 **(C)** 50 **(D)** 100

19. The sum of series $\cot^{-1}\left(\frac{9}{2}\right)+\cot^{-1}\left(\frac{33}{4}\right)+\cot^{-1}\left(\frac{129}{8}\right)+...\infty$

(A) $\cot^{-1}(2)$ **(B)** $\cot^{-1}(3)$ **(C)** $\cot^{-1}(-1)$ **(D)** $\cot^{-1}(1)$

20. If the function $f:[1,\infty)\to[1,\infty)$ is defined by $f(x)=2^{x(x-1)}$, then $f^{-1}(x)$ is :

(A) $\left(\frac{1}{2}\right)^{x(x-1)}$ **(B)** $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

(C) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$ **(D)** Not defined

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Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

1. Let $S=\left\{a:\log_2(9^{2a-4}+13)-\log_2\left(\frac{5}{2}\cdot 3^{2a-4}+1\right)=2\right\}$. Then the maximum value of β for which the equation $x^2-2\left(\sum_{a\in s}a\right)^2x+\sum_{a\in s}(a+1)^2\beta=0$ has real roots, is _____.

2. The remainder when 2^{2000} is divided by 17 is:

3. A triangle is formed by x-axis, y-axis and the line $3x+4y=60$. Then the number of points $P(a, b)$ which lie strictly inside the triangle, where a is an integer and b is a multiple of a, is _____.

4. If p and q be the longest and the shortest distances respectively of the point (–7, 2) from any point (α, β) on the circle $x^2+y^2-10x-14y-51=0$. If s is the geometric mean of p and q, then $\frac{s}{\sqrt{11}}$ is equal to

5. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is _____.

6. Let S be the sample space of all 3 × 3 matrices with entries from the set {0, 1}. Let the events E_1 and E_2 be given by

$E_1=\{A\in S:\det A=0\}$ and $E_2=\{A\in S:\text{sum of entries of } A \text{ is } 7\}$

If a matrix is chosen at random from S, then the conditional probability $10\times P(E_1/E_2)$ equals .....

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7. If $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\log_e a$ then $\frac{dy}{dx}=\frac{k\cdot y}{x}$, then k is equal to (where $a\in(0,\infty)$ is real constant)

8. Let f be a continuous function on R such that $f\left(\frac{1}{4n}\right)=(\sin e^n)e^{-n^2}+\frac{n^2}{n^2+1}$. Then the value of $f(0)$ is:

9. Let $f(x)=\min\left(\{x\},\{-x\}\right)\forall x\in R,$ where {.} denotes the fractional part of x, then $\int_{-100}^{100} f(x)\,dx$ is equal to:

10. Let there be three independent events E_1, E_2 and E_3 . The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ . Let 'p' denote the probability of none of events occurs that satisfies the equations $(\alpha-2\beta)p=\alpha\beta$ and $(\beta-3\gamma)p=2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to_____.

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